

# On the Optimality of Motion-Based Particle Filtering

Nidhal Bouaynaya, *Member, IEEE*, and Dan Schonfeld, *Senior Member, IEEE*

**Abstract**—Particle filters have revolutionized object tracking in video sequences. The conventional Particle filter, also called the CONDENSATION filter, uses the state transition distribution as the proposal distribution, from which the particles are drawn at each iteration. However, the transition distribution does not take into account the current observations, and thus many particles can be wasted in low likelihood regions. One of the most popular methods to improve the performance of particle filters relied on the motion-based proposal density first introduced in [1]. Although the motivation for motion-based particle filters could be explained on an intuitive level, up until now a mathematical rationale for the improved performance of motion-based particle filters has not been presented. In this paper, we investigate the performance of motion-based particle filters and provide an analytical justification of their superiority over the classical CONDENSATION filter. We rely on the characterization of the optimal proposal density, which minimizes the variance of the particles' weights. However, this density does not admit an analytical expression, making direct sampling from this optimal distribution impossible. We use the Kullback-Leibler (KL) divergence as a similarity measure between density functions and denote a particle filter as superior if the KL divergence between its proposal and the optimal proposal function is lower. We subsequently prove that under mild conditions on the estimated motion vector, the motion-based particle filter outperforms the CONDENSATION filter, in terms of the KL performance measure. Simulation results are presented to support the theoretical analysis.

**Index Terms**—Video tracking, particle filtering, motion estimation, adaptive block matching, Kullback-Leibler divergence.

## I. INTRODUCTION

Recently, Sequential Monte Carlo (SMC) filters [2] have become very popular for object tracking due to their flexibility and ease of implementation. These filters have been introduced to the tracking community in a paper by Isard and Blake [3]. SMC filters, also known as particle filters, are Bayesian filters, which use the Importance Sampling technique to estimate the distribution of non-linear and non-Gaussian state space models [2]. In the Bayesian framework, the tracking problem is formulated as the estimation of the posterior density of the target. In practice, the target's posterior is multimodal due to background clutter (hence non-Gaussian). Therefore,

traditional density estimation techniques (e.g., Kalman filter) will not apply. In Importance Sampling, a proposal density, also called importance function, is used to easily generate samples. Each sample is then assigned a proper weight to make up the difference between the posterior and proposal density functions [2]. It can be shown that if the number of samples is sufficiently large, the sample approximation of the posterior density can be made arbitrarily accurate [2]. However, in practice, only a finite number of samples can be used. Moreover, when a good dynamic model (i.e., a model that accurately predicts the target's dynamics) is not available, or the state dimension of the tracked object is high, the number of required samples becomes even larger and particle filtering can be computationally prohibitive. The conventional approach is to sample from image regions having high probability mass values. In [1], we proposed the use of a motion estimation algorithm to estimate the target position at each frame and draw samples from the area around the estimated position. Several advantages emerge of this choice. First, the motion is estimated on-line and hence no off-line motion learning is needed. Second, the tracker is adaptive to all kinds of motion. Particularly it can be viewed as a general case of the switching state space model technique [4]. Third, we economize on the processing time by searching only the state space around the estimated position. Finally, unlike a color or shape based proposal densities, a motion-based proposal is less sensitive to clutter. The incorporation of motion cues in particle filters have also been investigated in [5] and [6]. In [5], the authors proposed an adaptive state transition model by using an adaptive velocity model. In [6], an audio-visual tracking system is considered, where motion cues were incorporated in the likelihood model. The target's motion was estimated based on the absolute frame difference computed on successive pairs of images. Observe that, in [5], [6] and [1], motion information was incorporated in the state dynamics distribution, the likelihood model, and the proposal distribution, respectively.

Many proposal distributions have been proposed in the literature so far [7], where each distribution has been experimentally shown to outperform the conventional particle filter (or CONDENSATION). However, no one up till now tried to analyze analytically the performance of certain proposals over others. Zaritskii et. al. [8] derived the optimal proposal density,  $q_{\text{opt}}$ , which minimizes the variance of the particles' weights. However, this density does not admit an analytical expression in the general case, making direct sampling from it impossible. The challenge in particle filtering applications is, therefore, to design efficient proposal distributions that approximate the

N. Bouaynaya is with the Department of Systems Engineering, University of Arkansas at Little Rock, Little Rock, AR, 72204. e-mail: nxbouaynaya@ualr.edu

D. Schonfeld is with the Department of Electrical and Computer Engineering, University of Illinois at Chicago, Chicago, IL 60607. e-mail: dans@uic.edu

Copyright (c) 2008 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending an email to pubs-permissions@ieee.org.

optimal density as closely as possible. Although we cannot practically sample from the optimal proposal, we propose to assess the performance of a given proposal density,  $q$ , by computing the Kullback-Leibler (KL) “distance” or similarity measure between  $q_{\text{opt}}$  and  $q$ .

The remainder of this paper is organized as follows: In Section II, we review the motion-based particle filter introduced in [1], [9]. In Section III, we formulate a general optimization problem, based on the KL divergence, to assess the performance of different proposal densities compared to the optimal proposal, that minimizes the variance of the weights. We subsequently prove that, under mild conditions on the estimated motion vector (namely that it should not be the output of a random process but the output of a “decent” motion estimation algorithm), the motion-based particle filter is superior to the CONDENSATION filter, in the KL divergence sense. In Section IV, we present some simulation results. Finally, concluding remarks are discussed in Section V.

## II. MOTION-BASED PARTICLE FILTER

### A. Bayesian Sequential Importance Sampling

In Bayesian Sequential Estimation, a Markovian discrete-time state space model is assumed. Let  $X_k$  represent the target characteristics at discrete time  $k$  (position, velocity, shape, etc). The state space model is described by a state transition and measurement equations:

$$\begin{aligned} X_k &= f_k(X_{k-1}, v_k) \\ Z_k &= h_k(X_k, w_k), \end{aligned} \quad (1)$$

where  $f_k : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_x}$  and  $h_k : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_z}$  are possible non-linear functions. The stochastic processes  $v_k$  and  $w_k$  represent the state and measurement noise processes. Denote all past observations up to time  $k$  by  $Z_{1:k} = \{z_1, \dots, z_k\}$ . When the functions  $f_k$  and  $h_k$  are linear and the noise is gaussian, Kalman filter [10] provides an analytical closed form solution to the posterior density  $p(X_k|Z_{1:k})$ . However, for real world applications, the clutter introduces multiple observations and a multi-model density is then necessary to fairly model the posterior density.

In particle filtering, the posterior distribution is approximated by a set of weighted samples (also called particles) denoted by  $X^{(i)}$ .

$$p(X_k|Z_{1:k}) \approx \sum_{i=1}^N \pi_k^i \delta(X_k - X_k^{(i)}), \quad (2)$$

where  $\pi_k^{(i)} = \omega_k^{(i)} / \sum_{j=1}^N \omega_k^{(j)}$  is the normalized weight, and  $\omega_k^{(i)}$  is given by

$$\omega_k^{(n)} = \omega_{k-1}^{(n)} \frac{p(X_k^{(n)}|X_{k-1}^{(n)})}{q(X_k^{(n)}|X_{k-1}^{(n)}, Z_k)} p(Z_k|X_k^{(n)}). \quad (3)$$

Given a discrete approximation to the posterior distribution, one can then proceed to a filtered point estimate such as the mean of the state at time  $k$ ,

$$\hat{X}_k = \sum_{i=1}^N \pi_k^{(i)} X_k^{(i)}. \quad (4)$$

### B. Motion-Based Importance Function

Unlike shape and/or color based importance functions [11], [12] we proposed in [1] a motion-based importance density. Any motion estimation technique can actually be used in our algorithm. In case of compressed videos, using standards like MPEG, motion vectors are readily available. In our simulations, we choose the Adaptive Block Matching method (ABM) [13] because of its simplicity in implementation. For head tracking applications, we use a four dimension parametric ellipse to represent the state vector of the object:

$$X = [x_c, y_c, b, \phi]^T, \quad (5)$$

where  $(x_c, y_c)$  is the center of the ellipse,  $b$  is the minor axis of the ellipse, and  $\phi$  is the orientation of the ellipse in radians. The ratio of the major and minor axis of the ellipse is kept constant equal to its value computed in the first frame. For the experiments we conduct on head tracking, we found this assumption very reasonable and allows us to reduce the dimensionality of the state vector by 1. We use the Least Mean Square (LMS) criterion to fit the ellipse to the mask output of the ABM. The motion vector,  $\widehat{\Delta X}_k$ , that interests us is then given by the difference between the position of the center of the newly fitted ellipse  $(x_c(k), y_c(k))$  and the position of the center of the previous mean estimate  $(x_c(k-1), y_c(k-1))$ , i.e.,

$$\widehat{\Delta X}_k = [x_c(k) - x_c(k-1), y_c(k) - y_c(k-1), 0, 0], \quad (6)$$

The scaling and rotation parameters of each sample  $X_k^{(n)}$  (given by the 3<sup>rd</sup> and 4<sup>th</sup> coordinates) are randomly selected from a normal distribution with mean the scaling and rotation parameters of the previous sample  $X_{k-1}^{(n)}$  and fixed variances,  $\sigma_s^2$  and  $\sigma_r^2$  respectively.

We then have the following sampling scheme

$$X_k^{(n)} = X_{k-1}^{(n)} + \widehat{\Delta X}_k + v_k^{(n)}, \quad n = 1, \dots, N, \quad (7)$$

where  $v_k^{(n)}$  is the diffusion noise of sample  $n$  at time  $k$ .

Let  $\Delta X_k$  denote the true motion vector at time  $k$ . Then, we can write

$$\Delta X_k = \widehat{\Delta X}_k - v_k'', \quad (8)$$

where  $v_k''$  is the motion estimation error vector at time  $k$ . It is reasonable to assume that the motion error vector,  $v_k''$ , and the sampling noise,  $v_k^{(n)}$ , are uncorrelated for each sample  $n$  and at each time index  $k$ . Let  $v_k^{(n)}$  be the cumulative noise of the diffusion and the error in the motion estimation, i.e.,  $v_k^{(n)} = v_k^{(n)} + v_k''$ . Therefore, equation (7) becomes

$$X_k^{(n)} = X_{k-1}^{(n)} + \Delta X_k + v_k^{(n)}, \quad n = 1, \dots, N. \quad (9)$$

The reason behind expressing the sampling scheme in terms of the true motion vector  $\Delta X_k$  instead of the estimated motion vector  $\widehat{\Delta X}_k$  will become evident in Section III when we will compare the performance of the motion-based particle filter (MBPF) with the CONDENSATION filter. We assume that the vectors  $v_k''$  and  $v_k^{(n)}$  are normally distributed with zero mean and fixed covariance matrices. Thus, the cumulative noise vector

$v_k$  is also normally distributed with zero mean and covariance matrix  $\Sigma_G$ :

$$v_k \sim N(0, \Sigma_G), \quad (10)$$

and

$$\Sigma_G = \begin{pmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_s^2 & 0 \\ 0 & 0 & 0 & \sigma_r^2 \end{pmatrix}, \quad (11)$$

where  $\sigma_x^2$  and  $\sigma_y^2$  are the variances of the motion in the (x-y) direction. The motion-based importance density,  $q_m$ , is then the mixture of the translated and diffused samples around the target's estimated position, i.e.,

$$q_m(X_k|X_{k-1}, Z_k) \equiv \frac{1}{N} \sum_{n=1}^N \mathcal{N}_{X_k^{(n)}}(X_{k-1}^{(n)} + \Delta X_k, \Sigma_G), \quad (12)$$

where we made use of the following notation that we will be also using for the rest of this paper

$$\mathcal{N}_X(\mu, \Sigma) \equiv \frac{1}{2\pi|\Sigma|} \exp\left(-\frac{1}{2}(X - \mu)^T \Sigma^{-1}(X - \mu)\right). \quad (13)$$

In addition to the assessment of the importance function, we need to evaluate the likelihood,  $p(Z_k|X_k)$ , and the state dynamics,  $p(X_k|X_{k-1})$ , distributions (see Eq. (3)). We calculate the particle likelihood based on color [12] and edge [3] cues. We choose a random walk model for the system dynamics, which reflects a poor prior knowledge of the target's dynamics in the video [5], [6].

### III. OPTIMAL REALIZABLE PROPOSAL DENSITY

After synthesizing the proposed motion-based particle filter, we need to analyze its performance. The following basic question arises: What variables should we consider and what performance measure should we use? To study these questions, we consider the degeneracy problem of the Importance Sampling technique, where the variance of the particle's weights can only increase (stochastically) over time. That is most of the weight will be concentrated on a single particle after few iterations [2]. In practice, it is necessary to resample the particles to avoid degeneracy of the weights. Zaritskii et al. [8] introduced the optimal importance function which minimizes the variance of the weights. It is given by

$$q_{opt} = p(X_k|X_{k-1}^{(i)}, Z_k). \quad (14)$$

However, this optimal density does not admit an analytical expression in the general case, making direct sampling from it impossible. Though we cannot practically sample from  $q_{opt}$ , we can assess the performance of a proposal density  $q$  by computing some kind of "distance" or similarity measure between  $q_{opt}$  and  $q$ . The KL divergence,  $I(q_{opt}, q)$ , captures the "information" lost when a given proposal density  $q$  is used to approximate the optimal proposal density  $q_{opt}$ . It is defined as the multiple integral:

$$I(q_{opt}, q) = \int_{\mathcal{S}} q_{opt}(x) \log\left(\frac{q_{opt}(x)}{q(x)}\right) dx, \quad (15)$$

where  $\log$  denotes the natural logarithm and  $\mathcal{S}$  is the region of integration. We want to minimize  $I(q_{opt}, q)$  over a space of realizable densities indexed by  $\mathbf{Q}$ . The general optimization problem can be formulated as follows:

$$q^* = \min_{q \in \mathbf{Q}} I(q_{opt}, q), \quad (16)$$

where the space  $\mathbf{Q}$  does not contain the optimal proposal density  $q_{opt}$ ; i.e., the trivial solution  $q^* = q_{opt}$  is precluded. In order to reduce the state space, we consider  $\mathbf{Q}$  to be the set containing only two densities  $\{q, q_c\}$ , where  $q$  is any realizable density function and  $q_c$  is the proposal density in the CONDENSATION filter; i.e.,

$$q_c(X_k|X_{k-1}^{(i)}) = p(X_k|X_{k-1}^{(i)}).$$

In this paper, we will illustrate the approach proposed for characterization of the optimal realizable proposal density by focusing on motion-based particle filters. However, it is important to note that this approach could be easily extended to comparison of the performance (in the KL divergence sense) of any pair of proposal density functions in particle filtering.

Assume the set  $\mathcal{S}$  contains the region of support of the optimal density  $q_{opt}$ . Let  $\Omega$  be the set of densities  $q$  satisfying the inequality  $\int_{\mathcal{S}} \frac{q_{opt} q_c}{q} dX_k \leq 1$ . The following proposition shows that the space  $\Omega$  contains proposal densities, which are closer to the optimal density than the prior  $q_c$ .

*Proposition 1:* If  $q \in \Omega$  then  $I(q_{opt}, q) \leq I(q_{opt}, q_c)$ .

*Proof:* Let  $q$  be a realizable density such that  $q \in \Omega$ . Then we have

$$\begin{aligned} & I(q_{opt}, q) - I(q_{opt}, q_c) \\ &= \int_{\mathcal{S}} q_{opt} \log\left(\frac{q_{opt}}{q}\right) dX_k - \int_{\mathcal{S}} q_{opt} \log\left(\frac{q_{opt}}{q_c}\right) dX_k \\ &= \int_{\mathcal{S}} q_{opt} \log\left(\frac{q_c}{q}\right) dX_k \\ &\leq \log\left(\int_{\mathcal{S}} \frac{q_{opt} q_c}{q} dX_k\right) \end{aligned} \quad (17)$$

$$\leq \int_{\mathcal{S}} \frac{q_{opt} q_c}{q} dX_k - 1 \leq 0, \quad (18)$$

where Eq. (17) uses Jensen's Inequality and Eq. (18) uses the inequality  $\log(x) \leq x - 1$ . ■

Observe that every density  $q$  satisfying  $q \geq q_c$  belongs to the set  $\Omega$ , and therefore is closer to the optimal distribution than  $q_c$ . In the remainder of this section, we consider the prior to be a random walk with covariance matrix  $\Sigma_P$ , i.e.,

$$\begin{aligned} q_c = p(X_k|X_{k-1}) &= \frac{1}{N} \sum_{n=1}^N p(X_k^{(n)}|X_{k-1}^{(n)}) \\ &= \frac{1}{N} \sum_{n=1}^N \mathcal{N}_{X_k^{(n)}}(X_{k-1}^{(n)}, \Sigma_P). \end{aligned} \quad (19)$$

In the following proposition, we show that, under mild conditions on the target's motions,  $I(q_{opt}, q_m) \leq I(q_{opt}, q_c)$ .

*Proposition 2:* If  $\Sigma_G = \Sigma_P = \Sigma$  and  $(\frac{1}{2}\Delta X_k + v_k^{(n)})^T \Sigma^{-1} \Delta X_k \geq 0$ , for all  $1 \leq n \leq N$ , then  $I(q_{opt}, q_m) \leq I(q_{opt}, q_c)$ .

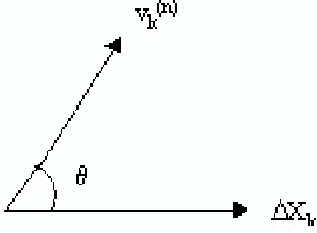


Fig. 1. A sufficient condition for the superiority of the motion-based proposal to the prior proposal, in terms of the KL divergence, is that the angle between the noise vector and the true motion vector does not exceed  $\frac{\pi}{2}$  in absolute value.

*Proof:* We show that  $q_m \geq q_c$ . From Eq. (12), we have

$$\begin{aligned} q_m(X_k|X_{k-1}, Z_k) &= \frac{1}{N} \sum_{n=1}^N \frac{1}{2\pi|\Sigma|} \\ &\exp\left(-\frac{1}{2}(X_k^{(n)} - X_{k-1}^{(n)} - \Delta X_k)^T \Sigma^{-1} (X_k^{(n)} - X_{k-1}^{(n)} - \Delta X_k)\right) \\ &= \frac{1}{N} \sum_{n=1}^N p(X_k^{(n)}|X_{k-1}^{(n)}) \exp\left(\left[\frac{1}{2}\Delta X_k + v_k^{(n)}\right]^T \Sigma^{-1} \Delta X_k\right) \quad (20) \\ &\geq \frac{1}{N} \sum_{n=1}^N p(X_k^{(n)}|X_{k-1}^{(n)}) = p(X_k|X_{k-1}) = q_c, \quad (21) \end{aligned}$$

where the equality in (20) follows from Eq. (19) and the inequality in (21) follows from the assumption that  $(\frac{1}{2}\Delta X_k + v_k^{(n)})^T \Sigma^{-1} \Delta X_k \geq 0$ , for all  $1 \leq n \leq N$ . ■

*Corollary 1:* If  $\Sigma_G = \Sigma_P = \Sigma$  and  $(v_k^{(n)})^T \Sigma^{-1} \Delta X_k \geq 0$ , for all  $1 \leq n \leq N$ , then  $I(q_{\text{opt}}, q_m) \leq I(q_{\text{opt}}, q_c)$ .

*Proof:* The proof follows immediately from Proposition 2 and since  $\Sigma^{-1}$  is a positive semi-definite matrix. ■

The condition in the corollary is equivalent to  $\frac{1}{\sigma_x^2} v_{k_x}^{(n)} \delta x_k + \frac{1}{\sigma_y^2} v_{k_y}^{(n)} \delta y_k \geq 0$ , where  $(v_{k_x}^{(n)}, v_{k_y}^{(n)})$  and  $(\delta x_k, \delta y_k)$  are the projections of the noise and the motion vectors along the first two coordinates, respectively. If  $\sigma_x = \sigma_y$ , we observe that a sufficient condition for the superiority of the motion-based proposal to the prior proposal, in terms of the KL divergence, is that the angle between the noise vector and the true motion vector does not exceed  $\frac{\pi}{2}$  in absolute value. This sufficient condition is illustrated in Figure 1.

Let us use  $|x|$  and  $\text{sign}(x)$  to denote the absolute value and sign of the scalar  $x$ , respectively. Moreover, we will use  $[\mathbf{x}]_i$  to denote the  $i^{\text{th}}$  component of the vector  $\mathbf{x}$  and  $[A]_{i,j}$  to represent the  $i, j^{\text{th}}$  element of the matrix  $A$ .

*Corollary 2:* If  $\Sigma_G = \Sigma_P = \Sigma$  and  $|\Delta X_k]_i| \geq 2|v_k^{(n)}]_i|$ , for  $i = 1, 2$ , and for every  $1 \leq n \leq N$ , then  $I(q_{\text{opt}}, q_m) \leq I(q_{\text{opt}}, q_c)$ .

*Proof:* The proof follows immediately from Proposition 2 and since  $\text{sign}([1/2\Delta X_k + v_k^{(n)}]_i) = \text{sign}([\Delta X_k]_i)$ , for  $i = 1, 2$ , and for every  $1 \leq n \leq N$ , and the fact that the elements of  $\Sigma_G^{-1}$  are non-negative (i.e.,  $[\Sigma_G^{-1}]_{i,j} \geq 0$ , for every  $i, j$ ). ■

In this case, we observe that a sufficient condition for the superiority of the motion proposal to the prior proposal, in the

KL divergence sense, is that the noise is small in comparison to the true motion of the tracked object. From Section II-B, recall that the sampling noise  $v_k$  is the sum of the motion estimation error and the diffusion noise. In particular, corollary 2 states that if the motion estimation error is “small enough” then the MBPF is superior to the CONDENSATION filter in the KL divergence sense. This result is somehow intuitive.

#### IV. EXPERIMENTAL RESULTS

We address real-life tracking scenarios where the target undergoes fast motion in one video (Fig. 2) and erratic sudden motion in another (Fig. 3). We compare the tracking results of the MBPF with the CONDENSATION filter. In Figure 2, the target is jumping. At least two equations are needed to describe its dynamics: the negative acceleration phase during the takeoff and the positive acceleration phase during the landing. The model gets more complicated if we take into account the fact that the initial velocity varies from one jump to another. Figure 2(a) displays the tracking output of the MBPF, which accurately captures the target’s position. The prior density of the MBPF is a random walk process. The CONDENSATION filter’s tracking result, with a random walk proposal density, was so poor and completely lost the target after few frames, that we chose not to display it here. Instead, we display the output of the CONDENSATION filter using a more sophisticated dynamic model given by the Langevin process [14]. Moreover, the parameters of the Langevin process were tuned offline to lead to the tracking results shown in Fig. 2(b). For real-time experiments, one cannot afford different off-line procedures to properly adjust the parameters of the dynamic model. Despite a more sophisticated dynamic model, the CONDENSATION filter loses the target in most of the frames. On the other hand, despite a poor prior knowledge of the target dynamics (expressed by a random walk dynamic model), the MBPF accurately tracks the jumping target. The reason behind this is that the prior plays no role in locating the sample set since the particles are sampled from the motion-based proposal density and not from the prior density.

Figure 3 shows a tracking scenario where the target undergoes erratic and sudden movements. The person in the video walks routinely then suddenly bends down and then stands up again as if he wanted to avoid someone’s (or something’s) sight. The CONDENSATION filter loses the target during the fast phase of bending and standing up again, whereas the MBPF accurately tracks the target during all phases.

#### V. CONCLUSION

Up until now, different choices of proposal densities including motion-based proposal functions, were only shown to outperform the conventional CONDENSATION filter experimentally. In this paper, we provided an analytical justification of the superiority of motion-based particle filters over the classical CONDENSATION filter. We used the Kullback-Leibler (KL) measure to compare the similarity of different proposal functions to an optimal un-realizable proposal density, which minimizes the variance of the particle’s weights. We proved that, if the motion estimation error is “small

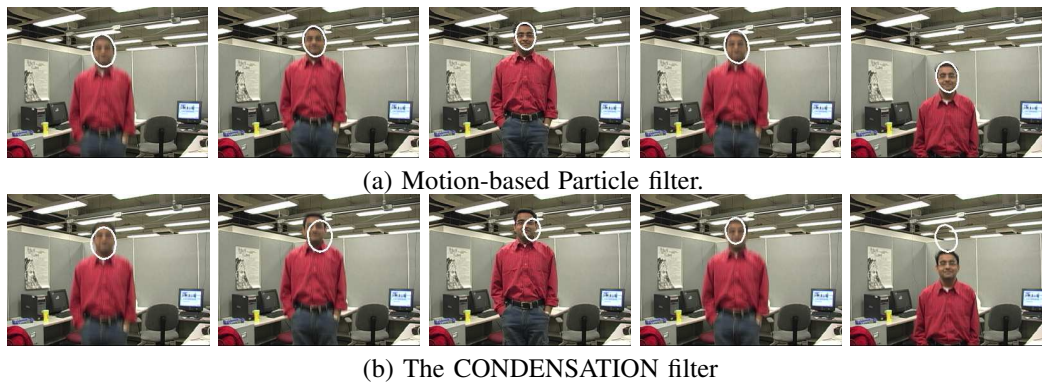


Fig. 2. Tracking a jumping person.

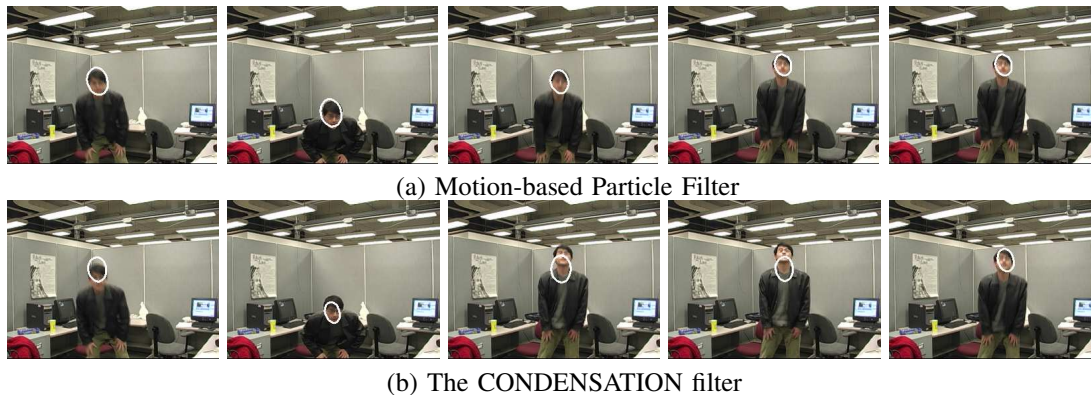


Fig. 3. Tracking sudden and erratic movements

enough” compared to the true motion, then the motion-based particle filter outperforms the CONDENSATION filter, in the KL divergence sense. We then presented simulation results to support our analysis. The proposed framework could be used for the analytical comparison of the various proposal functions used in particle filtering.

## REFERENCES

- [1] N. Bouaynaya, W. Qu, and D. Schonfeld, “An online motion-based particle filter for head tracking applications,” in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, March 2005, pp. 225–228.
- [2] A. Doucet, N. de Freitas, and N. Gordon, *Sequential Monte Carlo Methods in Practice*. New York: Springer-Verlag, 2001.
- [3] M. Isard and A. Blake, “CONDENSATION: conditional density propagation for visual tracking,” *International Journal of Computer Vision*, vol. 29, no. 1, pp. 5–28, 1998.
- [4] W. Zajdel, A. Cemgil, and B. Krose, “Online multicamera tracking with a switching state-space model,” in *Proceedings of IEEE International Conference on Pattern Recognition (ICPR)*, August 2004, pp. 23–26.
- [5] S. Zhou, R. Chellappa, and B. Moghaddam, “Visual tracking and recognition using appearance-adaptive models in particle filters,” *IEEE Transactions on Image Processing*, vol. 13, no. 11, pp. 1491–1506, November 2004.
- [6] P. Perez, J. Vermaak, and A. Blake, “Data fusion for visual tracking with particles,” *Proceedings of the IEEE*, vol. 92, no. 3, pp. 495–513, 2004.
- [7] S. Maskell, M. Briers, R. Wright, and P. Horridge, “Tracking using a radar and a problem specific proposal distribution in a particle filter,” *IEEE Proceedings*, vol. 152, no. 5, pp. 315–322, October 2005.
- [8] V. S. Zaritskii, V. B. Svetnik, and L. I. Shimelevich, “Monte carlo technique in problems of optimal data processing,” *Automation and Remote Control*, vol. 12, pp. 95–103, 1975.
- [9] N. Bouaynaya and D. Schonfeld, “Complete system for head tracking using motion-based particle filter and randomly perturbed active contour,” in *Proceedings of SPIE Image and Video Communications and Processing (IVCP)*, vol. 5685, San Jose CA, March 2005, pp. 864–873.
- [10] R. E. Kalman, “A new approach to linear filtering and prediction problems,” *Transactions of the ASME-Journal of Basic Engineering*, vol. 82, no. Series D, pp. 35–45, 1960.
- [11] M. Isard and A. Blake, “Icondensation: Unifying low-level and high-level tracking in a stochastic framework,” in *Proceedings of the European Conference on Computer Vision (ECCV)*, 1998, pp. 767–781.
- [12] Y. Wu and T. Huang, “Robust visual tracking by integrating multiple cues based on co-inference learning,” *International Journal of Computer Vision*, vol. 58, no. 1, pp. 55–71, 2004.
- [13] K. Hariharakrishnan and D. Schonfeld, “Fast object tracking using adaptive block matching,” *IEEE Transactions on Multimedia*, vol. 7, no. 5, pp. 853–859, 2005.
- [14] J. Vermaak and A. Blake, “Non linear filtering for speaker tracking in noisy and reverberant environments,” in *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 5, May 2001, pp. 3021–3024.